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Algebra-II

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Solving quadratic equations

$ax^2 + bx + c = 0$

1) factoring
2) quadratic formula
3) completing the square

Factoring

Perfect Squares:
$(a + b)^2 = a^2 + 2ab + b^2$

Difference of Squares:
$(a – b)^2 = a^2 – 2ab + b^2$

Sum of Cubes:
$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

Difference of Cubes:
$a^3 – b^3 = (a - b)(a^2 + ab + b^2)$

Factoring : when a =1

$ax^2 + bx + c = 0$

$ax^2 + 7x + 10 = 0$

When a = 1
1) Find two numbers
when multiplied = c &
when added = b
2 x 5 = 10
2 + 5 = 7
2) Write factors out
(x + 2) (x + 5) = 0
3) Set each factor = 0
x + 2 = 0; x + 5 = 0
4) Solve each factor
x = -2 ; x = -5

Factoring – when a is not equal to 1

$2x^2 – 7x – 15 = 0$

1) Find two numbers
when multiplied = a x c
in example (2) (–15) = –30 &
when added = b
(3) (-10) = –30 &
(3) + (-10) = –7
2) Replace b with new numbers
$2x^2 - 10x + 3x - 15 = 0$
3) Write as factors with parenthesis
$(2x^2 - 10x) + (3x – 15) = 0$
4) Factor GCF from each group
$2x(x – 5) + 3(x - 5) = 0$
5) Factor one more time
$(x – 5) + (2x + 3) = 0$
6) Set each factor = 0
x – 5 = 0; 2x + 3 = 0
7) Solve each factor
x = 5 ; x = –3/2

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$b^2 – 4ac > 0$, two real solutions
$b^2 – 4ac = 0$, one real solution
$b^2 – 4ac < 0$, no real solution

Completing the square

$ax^2 + 6x – 7 = 0$

a must = 1
1) Move c to right side
$x^2 + 6x = 7$
2) Divide b by 2 and square it
$(6/2)^2 = 9$
3) Add this number to both sides
$x^2 + 6x + 9 = 7 + 9$
4) Factor the left side
$(x + 3)(x + 3) = 16$
5) Rewrite left side as perfect square
$(x + 3)^2 = 16$
6) Take square root of both sides
$\sqrt{(x + 3)^2} = \pm \sqrt{16}$
$x + 3 = \pm 4$
7) Solve for x
x = 1 and -7
DONE

Completing the square – part 2

$2x^2 + 20x – 48 = 0$

if a is not = 1, you will factor
1) Move c to right side
$2x^2 + 20x = 48$
2) Factor the 2 out on left side
$2(x^2 + 10x) = 48$
3) Divide b by 2 and square it
$(10/2)^2 = 25$
4) Add this number to both sides
$2(x^2 + 10x + 25) = 48 + (2x25)$
5) Factor the left side
$2(x + 5)(x + 5) = 98$
6) Rewrite left side as perfect square
$2(x + 5)^2 = 98$
7) Isolate the square term
$(x + 5)^2 = 49$
8) Take square root of both sides
$\sqrt{(x + 5)^2} = \pm\sqrt{49}$
$x + 5 = \pm 7$
9) Solve for x
x = 2 and –12 DONE

Exponents

Multiplication – add exponents
$50^3 \times 50^4 = 50^7$

Division – subtract exponents
$\frac{50^6}{50^4} = 50^2$

Exponent to an exponent – multiply
$(50^6)^3 = 50^{18}$

Exponent to parenthesis
$(15 \times 8)^3 = 15^3 \times 8^3$
$\left(\frac{15}{8}\right)^3 = \frac{15^3}{8^3}$

Negative Exponent
$25^{-2} = \frac{1}{25^2} = \frac{1}{625}$

Zero Exponent - always equals 1
Any number to the zero = 1
for example: $345^0 = 1$

Roots

Addition
$3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$

Subtraction
$7\sqrt{5} - 4\sqrt{5} = 3\sqrt{5}$

Product
$\sqrt[2]{15 \times 25} = \sqrt[2]{15} \times \sqrt[2]{25}$

Quotient $\sqrt[4]{\frac{15}{25}} = \frac{\sqrt[4]{15}}{\sqrt[4]{25}}$

Root of a root
$\sqrt[3]{\sqrt[5]{25}} = \sqrt[3\times5]{25} = \sqrt[15]{25}$

Logarithms

Definition: $\log_2 5 = n$ means $2^n = 5$

Definition: $\log_a(a^n) = n$

Definition: $a^{\log_a b} = b$

Log of 1: $\log_a 1 = 0$

Log of the base: $\log_a a = 1$

Log of product: $\log_a(bc) = \log_a b + \log_a c$

Log of quotient: $\log_a(b/c) = \log_a b – \log_a c$

Log of reciprocal: $\log_a(1/c) = – \log_a c$

Log of power: $\log_a b^n = n\log_a b$

Change of base: $\log_a b = \log b \div \log a$

Graphing circles

Square

General equation of a circle

$(x - x_c)^2 + (y - y_c)^2 = r^2$

Example: $(x-1)^2 + (y+2)^2 = 3^2$

center: (1, –2) radius: 3

1) Find center: c = (1,–2
2) Find radius: r = 3
3) Graph point (1, – 2)
4) Go out 3 units in all directions

Graphing Quadratic / parabolas-I

$y = +/- x^2 + 8x + 15$

Opens up Opens down

1) find axis of symmetry
   x = -b/(2a)
   x = -8/2(1)= -4
2) find vertex by plugging axis of symmetry into equation to get y.
   $y = (-4)^2 + 8(-4) + 15$
   y = -1
3) vertex is x & y from above
   V = (-4, -1)
4) find x-intercepts by solving
   $x^2 + 8x + 15 = 0$
   (x + 3) (x + 5) = 0
   x = -3 & x = -5
5) Focus is p units away from vertex
   p = 1/ (4a)
   p = 1/(4 x 1) = ¼
   Since vertex = (-4, -1)
   Focus = (-4, -1+1/4)
   = (-4, -3/4)
6) Directrix is p units away from vertex in opposite direction
   Since vertex = (-4, -1)
   directrix = -1- 1/4
   y = -5/4

Graphing quadratic equation - II

1) Use completing the square strategy to change $y = ax^2 + bx - c$ to vertex form $y = a(x-x_v)^2 + y_v$

opens down; vertex

2) for example $y = -2x^2 - 4x + 5$
   Becomes $y = -2(x - 1)^2 + 3$
3) vertex = (1, 3)
4) Find x-intercepts by solving
   $-2(x - 1)^2 + 3 = 0$
5) Find focus as in graphing part - I
6) Find directrix

Graphing hyperbola

General equation of a hyperbola

Center point

$$\frac{(x-x_c)^2}{a^2} - \frac{(y-y_c)^2}{b^2} = 1$$

left & right up & down

x is first - opens sideways

Example: $\frac{(x-1)^2}{4^2} - \frac{(y+2)^2}{3^2} = 1$

1) plot center: c = (1, –2)
2) Go 4 units to right / left from center
3) Go 3 units to up / down from center
4) Draw rectangle & diagonals
5) Draw hyperbola
   If x is first, opens sideways
   If y is first, opens up & down
6) Find distance from center to foci:
   $a^2 + b^2 = c^2$
   $4^2 + 3^2 = c^2$
   5 = c
7) Foci is 5 spaces left / right from center
8) Vertices are always on longest axis
9) co-vertices are always on shorter axis

Note: the only difference between ellipse and hyperbola is the **+** & **-** sign

Graphing quadratic equation - III

Sum and product of roots

Sum of roots $= \frac{-b}{c}$

Product of roots $= \frac{c}{a}$

Axis of symmetry $x = \frac{-b}{2a}$

Vertex

put x value from axis of symmetry back into equation to get y

Vertex Form

$Y = a(x-x_v)^2 + y_v$

vertex

Probalility

Permutation: $_nP_k = \frac{n!}{(n-k)!}$

Combination: $_nC_k = \frac{n!}{(n-k)!\ k!}$

Graphing ellipses

General equation of an ellipse

Center point

$$\frac{(x-x_c)^2}{a^2} + \frac{(y-y_c)^2}{b^2} = 1$$

left & right up & down

Example: $\frac{(x-1)^2}{4^2} + \frac{(y+2)^2}{5^2} = 1$

Note: a is always the larger number, in this case a = 5 & is under the y term

1) plot center: c = (1, -2)
2) Go 4 units to right / left from center
3) Go 5 units to up / down from center
4) Draw ellipse
5) Find distance from center to foci:
   $a^2 - b^2 = c^2$
   $5^2 - 4^2 = c^2$
   3 = c
6) Foci is 3 spaces left / right from center
7) Vertices are always on longest axis
8) co-vertices are always on shorter axis

Sequence and Series

Arithmetic Sequence

n^th term: $t_n = t_1 + (n-1)d$

Sum: $S_n = \frac{n(t_1 + t_n)}{2}$

Geometric Sequence

n^th term: $t_n = t_1 (r)^{n-1}$

Sum: $S_n = \frac{t_1(1-r^n)}{1-r}$

Infinite sum: $S_\infty = \frac{t_1}{1-r}$

Convergent if $|r| < 1$

Factoring/ foiling

Perfect Squares:
$(a + b)^2 = a^2 + 2ab + b^2$

Difference of Squares:
$(a - b)^2 = a^2 - 2ab + b^2$

Sum of Cubes:
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Cubes:
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$